Quadratic Function

Opening Definitions:

Algebraic Expression: A set of terms separated by "+, – or = " sign.

Ex.: $2x^3 - 5x^2 + x - 7$

Terms are presented by multiplication of "factors", like: $-5x^2$ in the example above.

Equation: An expression containing " = " sign.

Ex.: $x^3 - 2x^2 + x - 3 = 0$

Relation: An equation with more than one variable.

A relation can be a **Function**, means one or more value of x, can have only one value for y.

Ex.: $y = x^3 - x^2 + 3x - 2$ *"x" is the independent variable, "y" the dependent.*

If one value of x has more than one y, this is a **Relation** but not a function.

Ex.: $x = y^2 - 2$ *since:* $y^2 = x + 2$ *Then:* $y = \pm\sqrt{x+2}$ *which is 2 Function.*

Quadratic Identities

Perfect Square identities:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Difference of Squares identity:

$a^2 - b^2 = (a - b)(a + b)$ (a - b) and (a + b) are known as **conjugate** factors.

Quadratic Equation:

General Form: $ax^2 + bx + c = 0$

a, b, and c, are real numbers.

Solving quadratic equation:

1. **By factoring**: change it to: $a(x – h)(x – k) = 0$, in which the solutions are: $x = h$ and $x = k$

Case 1, Ex.: *If a = 1:* $x^2 - 3x – 10 = 0$

info@e-tutorpro.com Phone: 1 604 710-9602

Look for 2 numbers that their sum is "b": h + k = -3 and their product is "c": h.k = - 10,

In this example those numbers are: -5 and 2. Therefore: (x – 5)(x + 2) = 0, and roots are 5 and -2.

Case 2, Ex. 1: *If $a \neq 1$:* $-2x^2 + 4x + 6 = 0$

Step 1: Factor GCF if possible,

Step 2: then proceed as per as case 1: $-2(x^2 - 2x - 3) = 0$

h + k = -2, and h.k = - 3, so h = -3 and k = 1. Therefore: -2(x – 3)(x + 1) = 0, so roots are 1 and -3.

Case 2, Ex. 2: $x^2 - 3x = 0$

GCF is "x", $x(x - 3) = 0$ *therefor: x = 0 and 3 are the zeros or roots*

Case 3, $a \neq 1$, and: $b, c \neq 0$

Ex.: $2x^2 + X - 6 = 0$

In this case the sum is still b; h + k = 1, but the product will be ac; h.k = 2.(-6) = - 12. So h = -3 and k = 4

Break down the middle term (+1) into 2 numbers, in order to change the trinomial to a Quatronomial:

$2x^2 + 4x - 3x - 6 = 0$ *then factor it 2 by 2*

$2x(x + 2) - 3(x + 2) = (x + 2)(2x - 3)$, *therefor: x = -2 and* $\frac{3}{2}$

2. **Completing square**:

Change the general form to: $a(x-h)^2 = k$, then square root both sides.

Ex. 1: $2(x-2)^2 - 18 = 0$ $2(x-2)^2 = 18$ $(x-2)^2 = 9$ $x - 2 = \pm 3$, *Therefor: x = -1, 5*

Ex. 2: $-2x^2 + 10x - 3 = 0$ $-2(x^2 + 5x) = 3$ $x^2 + 5x = -3/2$

Use quadratic identity to change it into: $(x + 5/2)^2$ *by dividing b by 2, (5/2) then add* $(5/2)^2$ *to both sides in order to make the left side a perfect square:*

$x^2 + 5x + 25/4 = -3/2 + 25/4$ *Therefor:* $(x + 5/2)^2 = 19/4$ *then:* $x = \pm\frac{\sqrt{19}}{2} - \frac{5}{2}$

3. **Quadratic formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $\Delta = b^2 - 4ac$

Nature of roots: $\Delta < 0$: no real root, $\Delta = 0$: one real root, $\Delta > 0$: 2 real roots

Sum of roots: $S = -\frac{b}{a}$ Product of roots: $P = \frac{c}{a}$ Vertex, Max or min are at: $x = -\frac{b}{2a}$

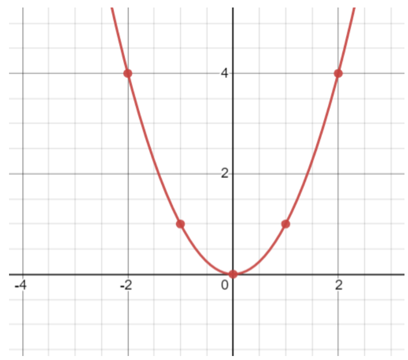
Quadratic Function:

General Form: $y = ax^2 + bx + c$

a, b, and c are real numbers.

The original form and the graph is called Parabola: $y = x^2$

$a \neq 0$, but b = 0 and c = 0



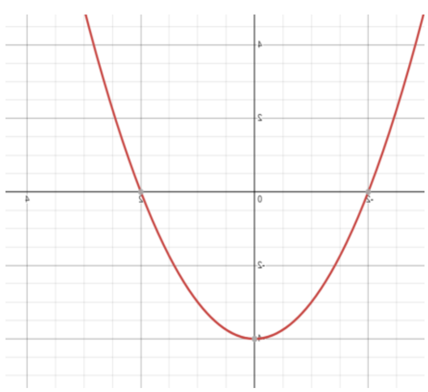
$a \neq 0$, but b = 0 and $c \neq 0$

$y = x^2 + c$

x = 0 is axis of symmetry

Vertex: V(0, c)

Ex. $y = x^2 - 4$

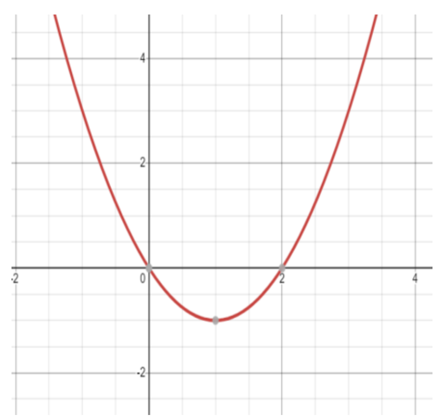


$a \neq 0$, but $b \neq 0$ and c = 0

$y = x^2 + bx = x(x - b)$

$x = \frac{-b}{2}$ is axis of symmetry

Ex. $y = x^2 - 2x$



Standard or Vertex form: $y = a(x - h)^2 + k$

Axis of symmetry: $x = h$, Vertex: V(h, k),

Factor form: $y = a(x - b)(x - c)$

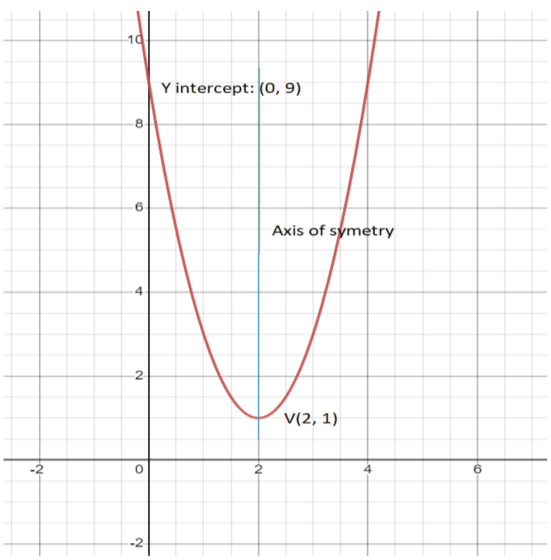
b and c are the x Intercepts, the axis of symmetry and the vertex at: $x = \frac{b+c}{2}$.

In all 3 cases "a" is the "Vertical Reflection, Expansion or Compression factor".

Examples of Quadratic functions and graphs:

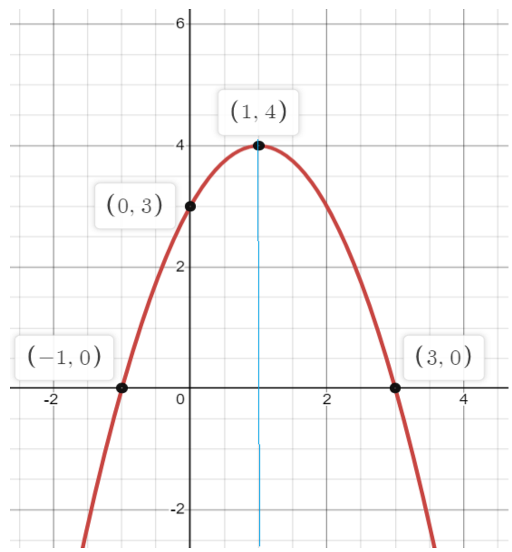
Vertex form: $y = 2(x-2)^2 + 1$

Vertex at: V(2, 1), Axis of symmetry: x = 2
Y intercept: (0, 9),
a = 2, vertically expanded by a factor of 2,
(positive, opening up)
$\Delta < 0$, No real root, no x intercept.



Factor form: $Y = -(x + 1)(x - 3)$

X Intercepts: -1, 3 y = 0
Y Intercept: - (-1).(3) = 3 x = 0
Axis of symmetry: x = 1 (-1 + 3)/2 = 1
Vertex at: V(1, 4) x = 1, y = 4
a = -1, (negative, opening down)



info@e-tutorpro.com
Phone: 1 604 710-9602