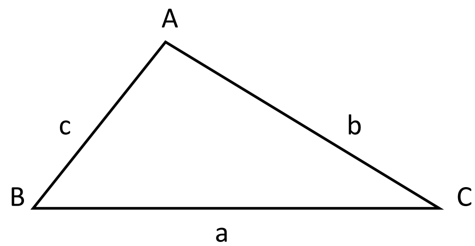
Solving non-right angle triangles:



A triangle is known with 6 numbers in total: 3 sides, 3 angles. Only 3 of them are enough to construct, or to graph a triangle by using “sine and cosine law” but at least side must be given:

Sine Law:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Cosine Law:

$$c^2 = a^2 + b^2 - 2ab.\cos C$$

This, calculates side “c”. To determine other sides, just alternate letters.

Case 1. Three sides a, b, c (sss), are given. Find angles A, B, C.

Using “Cosine law”, we can find any angles:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bC}$$

The second angle can be calculated the same way, but it’s easier to use “Sine law”:

$$\sin B = \frac{bx \sin A}{a} \qquad B = sin^{-1}(\frac{bx \sin A}{a})$$

The third angle is found by: C = 180° – (A + B). Only one possible solution.

Case 2. Given 2 sides “a, and b” and the angle in between “C” (sAs). Find the side c by using cosine law:

$$c^2 = a^2 + b^2 - 2ab \times \cos C$$

Then calculate angle A using Sine law:

info@e-tutorpro.com Phone: 1 604 710-9602

$$\sin A = \frac{a \times \sin C}{c} \qquad \text{Therefor:} \qquad A = sin^{-1}\left(\frac{a \times \sin C}{c}\right)$$

The third angle B = 180° – (A + C). Only one possible solution.

Those two cases above can only be solved by Cosine Law.

Case 3. Two angles and one side (AsA, AAs) are given. The third angle is calculated by the formula:

A+ B+ C = 180°, so the two cases become the same. Use the sines law to find the two other sides. Only one possible solution.

Case 4. Given two sides a, b and the angle B, opposite one of them (ssA), find side c and angles A and C.

At first by the sine law, find angle A:

$$\sin A = \frac{a \times \sin B}{b} \qquad \text{Therefor:} \qquad A = sin^{-1}\left(\frac{a \times \sin B}{b}\right)$$

The following 4 cases are possible:

1) a·sin B > b, which means, $\frac{a \times \sin B}{b} > 1$, this is not possible because: -1 $\leq sin\theta \leq 1$, so there is no solution. From the geometry point of view this means the side "a" is too short to touch side "b".

2) a·sin B = b, means: sinA = 1, therefor A = 90°, there is one solution, a right triangle.

3) a·sin B < b, means: sinA < 1, there are two solutions only if a > b, (ambiguous case). As we learn in trigonometry, for a given sine, we can always find two angles between (0 and 180°, in the first or second quadrant): So we can find two angle C_1 and C_2, therefor 2 different triangles.

We calculate first: $A_1 = sin^{-1}\left(\frac{a \times \sin B}{b}\right)$, which is an acute angle, and then; $A_2 = 180° – A_1$ will be an obtuse angle: $C_1 = 180 – B – A_1$, and $C_2 = 180 – B – A_2$.

Then calculate the side c for each pair of angles A and C using:

$$c = \frac{b \sin C}{\sin B}$$

4) a·sin B < b, means: sinA < 1, but a < b, in this case $A_2 + B > 180°$, so there is only one possible solution.

Find side "c" from the formula above.

info@e-tutorpro.com Phone: 1 604 710-9602